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Flow rate determination by cross correlation of temperature noise signals from out-of-stream thermocouples

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NOMENCLATURE

A	constant
$C_{ii}(\tau)$	auto correlation function
$C_{ij}(\tau)$	cross correlation function
H	transfer function
Re	Reynolds number
T	time [s]
a	thermal diffusivity [$m^2\ s^{-1}$]
f	frequency [s^{-1}]
t	time [s]
v	fluid velocity [$m\ s^{-1}$]
x	radial position inside the channel wall [m]
z	axial position along the flow channel [m].

Greek symbols

$\Phi_{ii}(\omega)$	autopower spectral density function
$\Phi_{ij}(\omega)$	cross power spectral density function
$\gamma_{ij}^2(\omega)$	coherence function
θ	temperature [K]
τ	time [s]
ϕ	phase
ω	angular frequency [s^{-1}].

INTRODUCTION

FLOW RATE determination is an important problem in a wide range of applications. Therefore quite a variety of methods and instruments have been developed for flow rate and local flow velocity measurements. In many cases detectors are to be placed in the liquid stream and will therefore influence the flow velocity more or less, for example turbine flowmeters, pitot pressure tubes, hot wire probes, etc. Especially in situations where parallel cooling channels with a rather small cross section are used, flow determination in one channel without influencing the flow distribution will be difficult. More sophisticated techniques with laser-Doppler systems, electrodynamic or inductive type transducers do not have this disadvantage, but these methods put higher demands on the experimental environment and are not always appropriate.

In systems like nuclear reactors determination of coolant flow in individual cooling channels is often impossible with instruments mentioned above. Safety considerations make a solution even more difficult. This paper describes an experimental method to determine the flow rate in cooling channels by measuring the propagation time of temperature fluctuations present in the liquid flow. Information about flow rate and flow velocity is usually important in case of heat transfer by forced convection. This cooling process will induce temperature fluctuations in the coolant, which can be measured by thermocouples positioned inside the channel wall material at a certain axial distance. The thermocouples will register in succession the influence of the fluctuating liquid temperature with a transit time depending on the flow velocity near the heated wall.

Determination of this transit time can be done by cross correlation of the noise component of the thermocouple signals. The application of out-of-stream thermocouples provides a measuring technique which is non-perturbing with respect to local flow velocity and flow rate distribution. The feasibility depends on sufficient coherence between the temperature noise signals from different thermocouples. This implies for example, that the axial distance between the couples must be chosen within a limited range. The damping of the temperature fluctuations during radial propagation in the channel wall limits the distance between the inside channel surface and the location of the thermocouples; this limitation is dependent on the thermal properties of the channel wall material.

The purpose of the work reported in this article was to investigate the feasibility of flow rate measurements with non-perturbing out-of-stream thermocouples, focusing on the type of velocity information obtained. It is well known that in-stream space–time correlation measurements lead to an apparent transfer velocity, deviating from the average fluid velocity, depending on the conditions in the bulk stream and the boundary layer. The physical background of this phenomenon has been studied extensively [2]; an analysis of the temperature fluctuations in the wall of a conduit is however lacking up to now. The work reported here had a purely empirical approach, to find out what type of flow rate information is obtained from space–time correlation of out-of-stream thermocouple signals in dependence of the flow conditions in the conduit.

EXPERIMENTAL APPARATUS

At the Interuniversity Reactor Institute in Delft an experimental rig has been constructed in order to perform boiling detection investigations in a test section which simulates a nuclear fuel element. The simulated fuel element will be positioned adjacent to a nuclear reactor core of a 2 MW 'pool' type research reactor and is equipped with thermocouples and neutron and gamma radiation detectors. In Fig. 1 a cross section of the element is shown. It contains three electrically heated aluminium plates with dimensions $625 \times 65 \times 4\ mm^3$; see also Fig. 2. Each plate consisted of two parts.

In one part grooves were made for a thermocoax heating wire (diameter 1.5 mm) and for five mineral insulated thermocouples (type J, diameter 0.5 mm). The two plate parts were joined with an explosive welding technique to achieve a perfect heat conducting junction over the whole contact area. A good thermal contact between thermocouples and heating wire with their environment is also created using this method.

The plates are heated with an electrical power supply of 10 kW and are cooled by demineralised water flowing upwards through four vertical cooling channels with cross sections of $3 \times 65\ mm^2$. These dimensions show that application of instream thermocouples will disturb the coolant flow and make the determination of the flow velocity unreliable. The hydraulic diameter of a cooling channel is 5.75 mm. The cooling system is independent from the reactor cooling, which enables the inlet coolant temperature, flow rate and channel power to be controlled, thus allowing one to create different cooling conditions with and without boiling phenomena in the simulated element. Information about the coolant flow in the

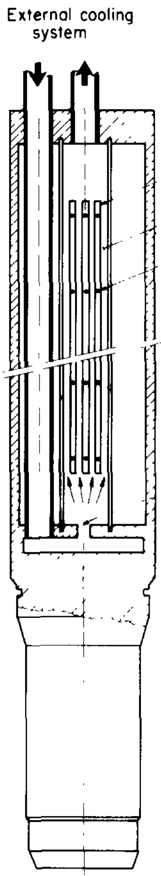

FIG. 1. Cross section of the simulated fuel element.

separate cooling channels will be important for future experiments with the element positioned near the reactor core.

During the experiments the coolant temperature at the inlet of the element was 50°C. Heating power was kept constant and outlet temperature ranged from 55 to 75°C depending on flow rate.

CROSS CORRELATION TECHNIQUE

In Fig. 3 a cooling channel is shown schematically. Temperature fluctuations in the cooling liquid, caused by the

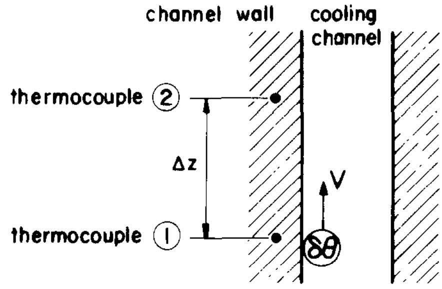

FIG. 3. Schematic representation of a cooling channel.

heat transfer process, will be detected first by thermocouple 1 and after a certain transit time detected by thermocouple 2. The thermocouple signals consist of a mean value $\langle V \rangle$, corresponding with a stationary temperature and a noise component $\delta V(t)$ superposed on the mean value and caused by the temperature fluctuations in the channel wall. These fluctuations are induced by a fluctuating coolant temperature. The relation for the thermocouple signals is

$$V(t) = \langle V \rangle + \delta V(t). \tag{1}$$

The dynamic component can be divided into a coherent part $\delta V_c(t)$, present in both signals but with a time shift (due to the fluid transport time), and an incoherent part $\delta V_i(t)$. This incoherent part contains the influence of temperature fluctuations, which are registered only by one of the two thermocouples and by background noise from electronic equipment like amplifiers, filters, cables, etc. The temperature noise signals can be written as

$$\delta\theta_1(t) = \delta\theta_c(t) + \delta\theta_{i,1}(t), \tag{2}$$

$$\delta\theta_2(t) = \delta\theta_c(t-\Delta t) + \delta\theta_{i,2}(t). \tag{3}$$

The autocorrelation functions (ACF) of these signals are respectively

$$C_{11}(\tau) = \lim_{T\to\infty} \frac{1}{T} \int_0^T \delta\theta_1(t)\delta\theta_1(t+\tau)\,dt \tag{4}$$

and

$$C_{22}(\tau) = \lim_{T\to\infty} \frac{1}{T} \int_0^T \delta\theta_2(t)\delta\theta_2(t+\tau)\,dt. \tag{5}$$

Autocorrelation functions always have a maximum for $\tau = 0$ [1].

The cross correlation function (CCF) of these two noise

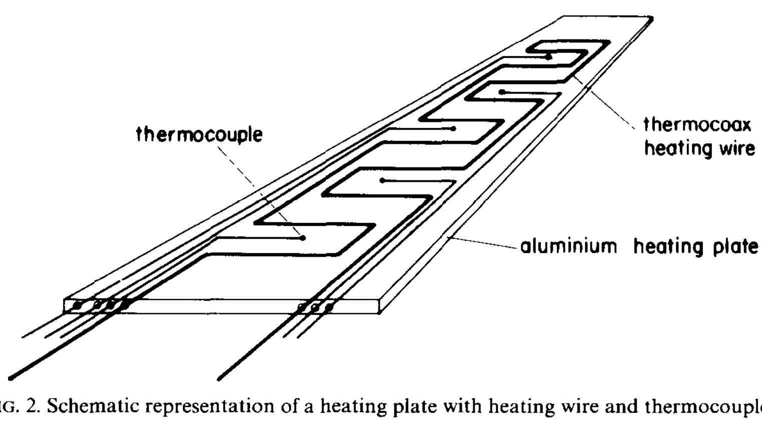

FIG. 2. Schematic representation of a heating plate with heating wire and thermocouples.

signals can be determined as follows

$$C_{12}(\tau) = \lim_{T\to\infty} \frac{1}{T}\int_0^T \delta\theta_1(t)\delta\theta_2(t+\tau)\,dt$$
$$= \lim_{T\to\infty} \frac{1}{T}\int_0^T \delta\theta_c(t)\delta\theta_c(t+\tau-\Delta t)\,dt$$
$$= C_{cc}(\tau-\Delta t), \quad (6)$$

because the terms containing incoherent parts vanish. C_{cc} is the autocorrelation function of the coherent temperature fluctuations.

According to equation (6) the transit time Δt can be determined, because the CCF will have a maximum for $\tau = \Delta t$. Only if the noise signals have a so called 'white' frequency spectrum can a sharp peak be noticed at $\tau = \Delta t$. In the case of the thermocouples the signal information exists mainly in the lower frequency range. A rounded peak will appear at $\tau = \Delta t$, which causes an inaccuracy in the determination of a transit time, see Fig. 4. A frequency dependent description will increase the accuracy and gives also more information about the registered noise signals, for example whether the time shift in two thermocouple signals is independent of the signal frequency or not.

Fourier transformation of the ACF and CCF gives respectively the auto power spectral density functions (APSD) and the cross power spectral density function (CPSD) of the two temperature noise signals [1]

$$\text{APSD} = \Phi_{11}(\omega) = \int_{-\infty}^{\infty} C_{11}(\tau)\,e^{-j\omega\tau}\,d\tau \quad (7)$$

$$\text{CPSD} = \Phi_{12}(\omega) = \int_{-\infty}^{\infty} C_{12}(\tau)\,e^{-j\omega\tau}\,d\tau$$
$$= \int_{-\infty}^{\infty} C_{cc}(\tau-\Delta t)\,e^{-j\omega\tau}\,d\tau$$
$$= e^{-j\omega\Delta t}\int_{-\infty}^{\infty} C_{cc}(\tau-\Delta t)\,e^{-j\omega(\tau-\Delta t)}\,d\tau$$
$$= e^{-j\omega\Delta t}\,\Phi_{cc}(\omega), \quad (8)$$

where $\Phi_{cc}(\omega)$ is the Fourier transform of C_{cc}.

Contrary to the APSD, the CPSD is a complex function of frequency and can be divided into a modulus and a phase relation

$$|\Phi_{12}(\omega)| = \Phi_{cc}(\omega), \quad (9)$$
$$\phi_{\text{CPSD}}(\omega) = -\omega\Delta t. \quad (10)$$

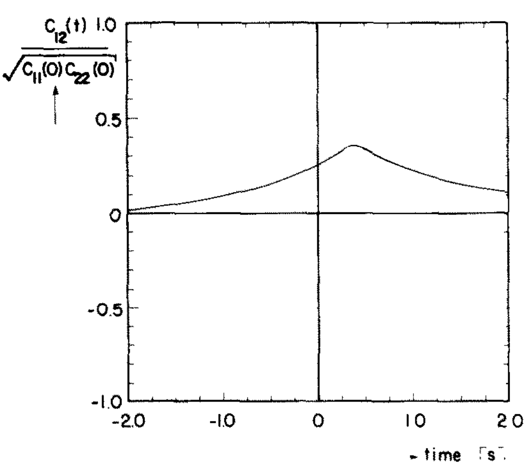

FIG. 4. Cross correlation function of two temperature noise signals.

In the case that the coherent part of the noise signal δV_c is sufficiently strong an accurate linear phase-to-frequency curve can be obtained, which enables calculation of a transit time with good accuracy.

The flow velocity v can be determined, by $v = \Delta z/\Delta t$, where Δz is the axial distance between the thermocouples. The coherence between the two noise signals can be calculated with the following relation [1]

$$\gamma_{12}^2(\omega) = \frac{|\Phi_{12}(\omega)|^2}{\Phi_{11}(\omega)\cdot\Phi_{22}(\omega)}. \quad (11)$$

A lower limit for an acceptable coherence depends on the bandwidth of the signals, the transit time itself and the accuracy that can be obtained in a certain measuring time.

EXPERIMENTAL RESULTS

Under different flow conditions thermocouple signals have been recorded on tape after amplification and filtering. Afterwards the signals were sampled by a computer controlled analysing system. With a Fast Fourier Transform algorithm a frequency-dependent description is obtained and the APSDs of the temperature noise signals and also the CPSDs and coherence functions of a certain signal combination can be calculated. For flow velocity determination thermocouples in two neighbouring plates at a certain axial distance have been used. Thermocouples in the same heating plate will be influenced by the coolant flow in two adjacent cooling channels and this complicates the phase relation.

In Fig. 5a autospectra of two temperature noise signals are plotted. From the shape of the APSDs it can be seen that the signal information is limited to the lower frequency range. The phase function of the CPSD plotted in Fig. 5b, shows a clearly linear relationship. The experimentally determined function is numerically fitted to a straight line using a least squares fit procedure. To examine the applicability of a linear relationship the χ^2 test is performed. The coherence function shown in Fig. 5c gives an impression about the contribution of the coherent part δV_c in the two thermocouple signals. In Fig. 6 the frequency functions of another thermocouple combination under other flow conditions are plotted. Even at an axial distance of 30 cm a transit time could be detected.

The determination of the flow velocity from the calculated phase function has an inaccuracy less than 1% after a measuring time of 5 min. To check the usefulness of this technique experiments were also performed with a similar rig, where the flow through the individual cooling channels could be adjusted and measured directly with rotameters.

The results of the calculations for one cooling channel are shown in Fig. 7. Along the abscissa the values obtained from the rotameters are drawn and along the ordinate the flow rate calculated with cross correlation of the temperature noise signals. The Reynolds numbers were calculated using values for viscosity and density at the average coolant temperature in the channel. Experiments at Reynolds numbers under 900 brought no useful information because the coherence between the two thermocouple signals was too small. At higher values of the Reynolds number ($Re > 2000$), as characteristic for most practical cooling conditions, there is quite a good agreement between both measuring techniques. In that case the velocity profile will be rather flat, contrary to situations with lower flow rates. Then the profile is curved by free convection near the channel walls, which has a relatively stronger influence at lower Reynolds numbers. To investigate this phenomenon experiments with an opposite flow direction are being prepared.

PROPAGATION OF TEMPERATURE FLUCTUATIONS IN THE CHANNEL WALL

Temperature fluctuations measured by thermocouples in the heated plates are caused by heat transfer and heat

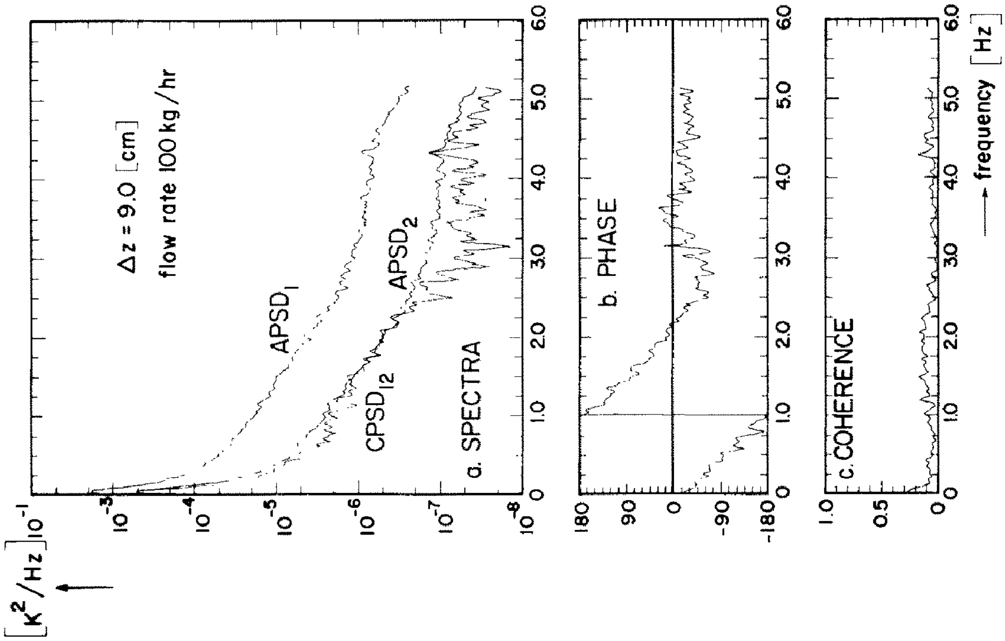

FIG. 6. Frequency functions of temperature noise signals.

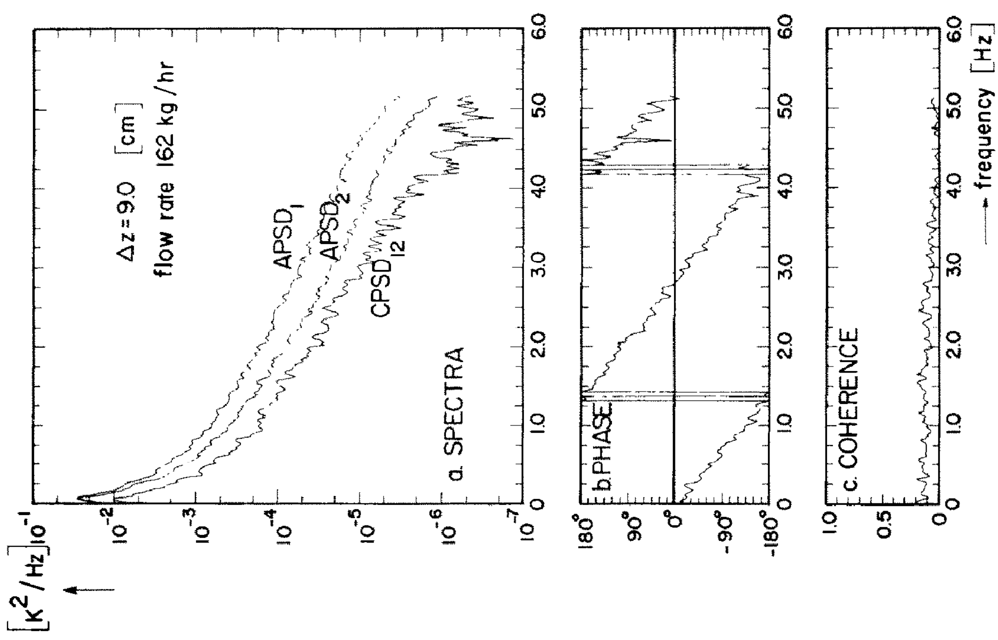

FIG. 5. Frequency functions of temperature noise signals.

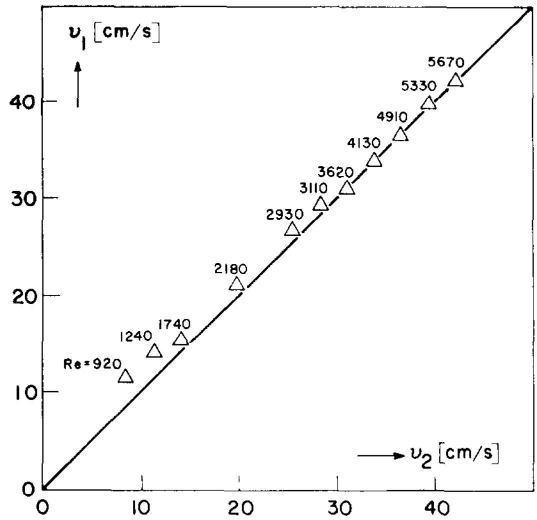

FIG. 7. Comparison of flow rate measured by a rotameter (v_2) with flow rate determined by cross correlation of temperature noise (v_1).

dissipation processes in the coolant flow. A laminar flow velocity profile as present in an isothermal flow at lower Reynolds numbers will be disturbed by these processes. The higher temperatures of the liquid near the heated channel walls cause a decrease of local viscosity and density and therefore an increase of the upward flow velocity. The velocity profile will be smoothed by momentum and energy exchange perpendicular to the flow direction. This exchange mechanism also causes fluctuations in the temperature and the flow velocity of the liquid near the channel walls and temperature fluctuations at the surface of the heating plates. To describe the *propagation of the temperature fluctuations through the* aluminium plate material a mathematical relation will be derived below.

The heating plate is considered as a half-infinite homogeneous body ($0 < x < \infty$) and the temperature fluctuations $\delta\theta(x,t)$ induced at $x = 0$ are supposed to propagate in one direction. For one dimension the temperature equation is

$$\frac{\partial}{\partial t}\,\delta\theta(x,t) = a\,\frac{\partial^2}{\partial x^2}\,\delta\theta(x,t) \tag{12}$$

with

$$\delta\theta(x,t) = \theta(x,t) - \langle\theta\rangle. \tag{13}$$

Fourier transformation of equation (12) gives:

$$j\omega\,\delta\theta(x,\omega) = a\,\frac{\partial^2}{\partial x^2}\,\delta\theta(x,\omega). \tag{14}$$

A boundary condition is that $\lim_{x\to\infty} \delta\theta(x,t)$ must be finite, while the fluctuations $\delta\theta(0,\omega)$ may have an arbitrary frequency spectrum. Solution of equation (14) leads to

$$\delta\theta(x,\omega) = \delta\theta(0,\omega)\exp[-(1+j)\sqrt{(\omega/2a)}x]. \tag{15}$$

A transfer function describing the influence of temperature fluctuations at the plate surface on the registered temperature noise of the thermocouples positioned in the heating plates at a depth x can be written as

$$H(x,\omega) = \frac{\delta\theta(x,\omega)}{\delta\theta(0,\omega)}, \tag{16}$$

with modulus

$$|H(x,\omega)| = \exp[-\sqrt{(\omega/2a)}x] \tag{17}$$

and phase

$$\phi_H(x,\omega) = -\sqrt{(\omega/2a)}x. \tag{18}$$

From equation (17) it can be concluded that temperature fluctuations can always be measured in a certain frequency range and at a given position x if the influence of the background noise (for example from electronic equipment) is not too strong.

In the aluminium heating plates ($a_{Al} = 9.5 \times 10^{-5}\ m^2\,s^{-1}$) the position of the thermocouples was 2 mm under the plate surface. If stainless steel was applied ($a = 4.4 \times 10^{-6}\ m^2\,s^{-1}$) a depth of 0.4 mm is required for flow velocity determination with the same accuracy.

To check the usefulness of this model measurements have been performed with a thermocouple positioned in the liquid flow. The hot junction was close to the plate surface at approximately the same axial position as a thermocouple inside the heating plate. The situation is shown in Fig. 8. In Fig. 9a the APSDs of the temperature noise measured by the two thermocouples are plotted. The RMS-value of the temperature noise in the liquid flow is considerably higher than in the heating plate. The phase function of the CPSD plotted in Fig. 9b is fitted to the function

$$\phi_{CPSD}(f) = \phi_H(f) = A_1\sqrt{f} + A_2 f. \tag{19}$$

The fitted curve with $A_1 = -72.2\ s^{1/2}$ and $A_2 = 3.05$ s is also plotted in Fig. 9b. The term $A_1\sqrt{f}$ comes from thermal conduction according to equation (18). The linear term $A_2 f$ describes the convective transport of temperature fluctuations in the cooling channel between the axial positions of the instream and out-of-stream thermocouples. Although the relative positions of the couples inside the heating plates are well known the axial distance z between the instream couple and the plate couple used during this experiment was only known with an accuracy of a few millimetres.

In an extended model we divide the transfer function H in two parts: one describing the convective transport in the coolant flow H_1, and one describing heat transport by conduction through the plate material H_2. Conductive transport through the channel wall not perpendicular to the flow direction (indicated as (3) in Fig. 10) is neglected in this model. The total transfer function is

$$H(\omega) = H_1(\omega)\cdot H_2(\omega)$$
$$= |H_1|\exp -j\omega(z/v)\cdot|H_2|\exp -j\sqrt{(\omega/2a)}x \tag{20}$$

with v the average flow velocity in the cooling channel.

The modulus of $H(\omega)$ cannot be calculated easily because $|H_1|$ is not known, but the phase function of $H(\omega)$ is described by a simple relation like equation (19), with

$$A_1 = -\frac{180x}{\sqrt{(a\cdot\pi)}}$$

and

$$A_2 = -360\tau = -360\frac{z}{v}.$$

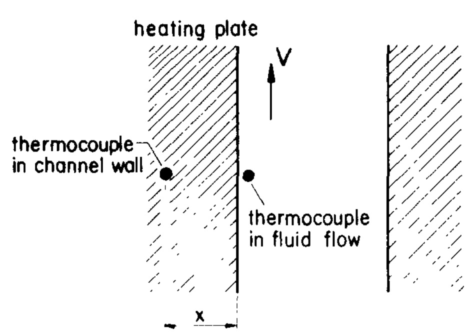

FIG. 8. Positions of instream and out-of-stream thermocouple.

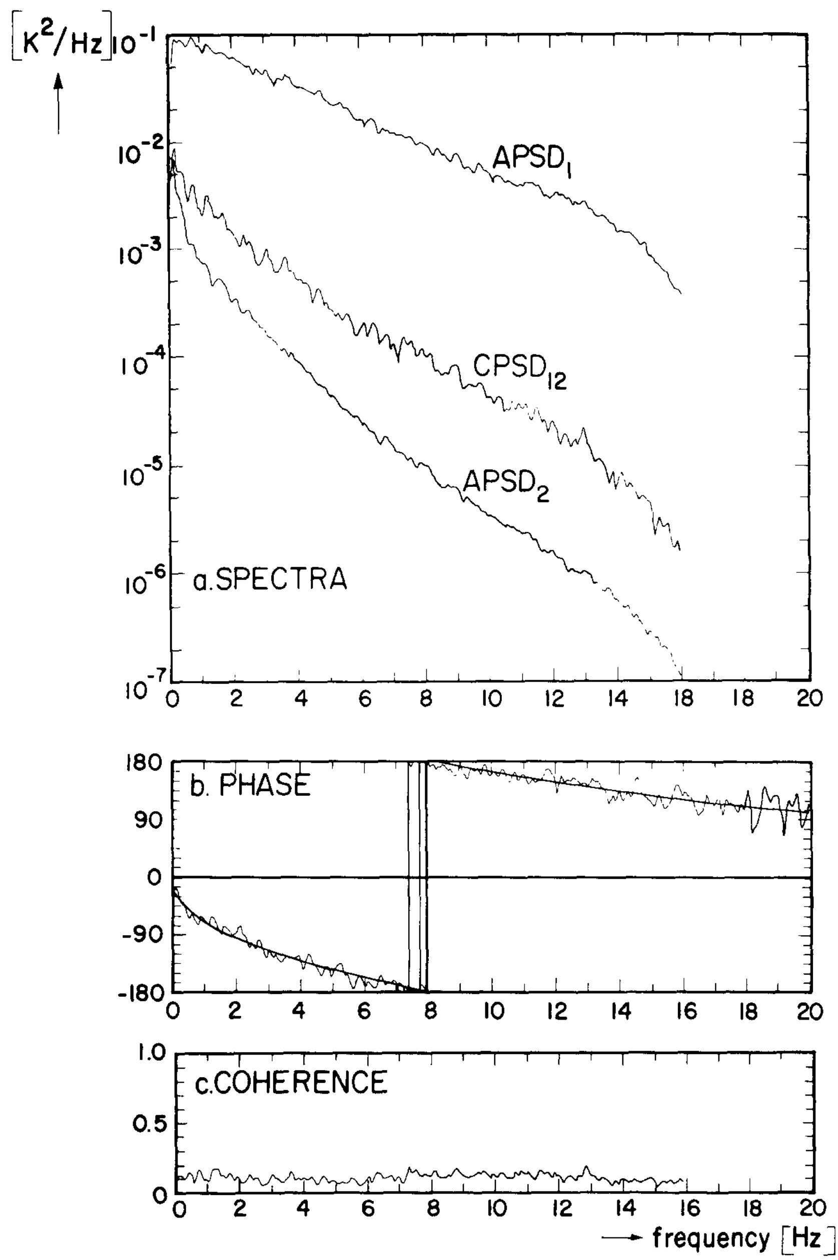

FIG. 9. Frequency functions for an instream and out-of-stream thermocouple.

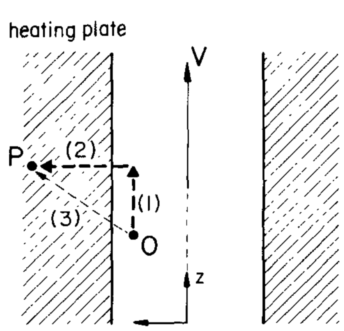

FIG. 10. Heat transport from position O in the liquid flow to position P in the channel wall.

From the fitted curve in Fig. 9b it can be shown that $x = 2.2(\pm 0.3)$ mm and $z = 2.0(\pm 0.2)$ mm. These values are quite acceptable with respect to the accuracy of the fitting procedure and the inaccuracy of the thermocouple position.

CONCLUSIONS

The application of cross correlation of temperature noise registered by out-of-stream thermocouples provides a useful method to determine flow velocity in cooling channels. The determination can be achieved with a relatively high accuracy at higher Reynolds numbers and without local disturbance of the velocity profile and flow rate distribution in case of parallel cooling channels.

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